

Progression in Calculation



Aims

The national curriculum for mathematics aims to ensure that all pupils:

- become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.
- **reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

Introduction

Written methods of calculations are based on mental strategies. Each of the four operations builds on mental skills which provide the foundation for jottings and informal written methods of recording. Skills need to be taught, practised and reviewed constantly. These skills lead on to more formal written methods of calculation.

Strategies for calculation need to be represented by models and images to support, develop and secure understanding. This, in turn, builds fluency. When teaching a new strategy it is important to start with numbers that the child can easily manipulate so that they can understand the methodology.

The transition between stages should not be hurried as not all children will be ready to move on to the next stage at the same time, therefore the progression in this document is outlined in stages. Previous stages may need to be revisited to consolidate understanding when introducing a new strategy.

A sound understanding of the number system is essential for children to carry out calculations efficiently and accurately.

Magnitude of Calculations

Year 1 – $U + U$, $U + TU$ (numbers up to 20), $U - U$, $TU - U$ (numbers up to 20), $U \times U$, $U \div U$

Year 2 - $TU + U$, $TU +$ multiples of 10, $TU + TU$, $U + U + U$, $TU - U$, $TU -$ tens, $TU - TU$, $TU \times U$, $U \div U$

Year 3 – add numbers with up to three-digits, $HTU +$ multiples of 10, $HTU +$ multiples of 100, subtract numbers up to three-digits, $HTU - U$, $HTU -$ multiples of 10, $HTU -$ multiples of 100, $HTU - HTU$, $TU \times U$, $TU \div U$

Year 4 - add and subtract numbers with up to four-digits, $ThHTU + ThHTU$, $ThHTU - ThHTU$, add and subtract decimals with up to two decimal places in the context of money, multiply three numbers together, $TU \times U$, $HTU \times U$, $TU \times U$, multiply by zero and one, $TU \div U$, $HTU \div U$

Year 5 – add and subtract numbers with more than four-digits, add and subtract decimals with up to three decimal places, $ThHTU \times U$, $ThHTU \times TU$, $HTU \times TU$, multiply whole numbers and decimals with up to three-decimal places by 10, 100 and 1000, divide numbers with up to four-digits by U (including remainders as fractions and decimals and rounding according to the context)

Year 6 - add and subtract numbers with more than four-digits, add and subtract decimals with up to three decimal places, multiply numbers with up to four-digits by TU , multiply numbers with up to two-decimal places by a whole number, divide numbers up to four-digits by TU (interpreting remainder according to the context), divide decimals up to two-decimal places by U or TU

Mathematics is an interconnected subject in which pupils need to be able to move fluently between representations of mathematical ideas. ... pupils should make rich connections across mathematical ideas to develop fluency, mathematical reasoning and competence in solving increasingly sophisticated problems. They should also apply their mathematical knowledge to science and other subjects.

National Curriculum 2014

Structuring Learning

Children must have concrete experiences that enable them to create visual images. They should be encouraged to articulate their learning and to become pattern spotters.

Language

Symbols

Pictures

Concrete Experiences

Haylock and
Cockburn (2008)

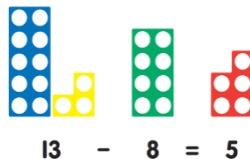
Active/concrete

Building visual images

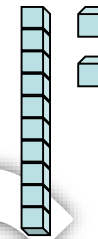
Abstract

Communicating
Mathematically

Pattern
Spotting

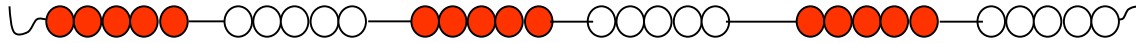


$$13 - 8$$



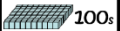


$$12 + 19$$

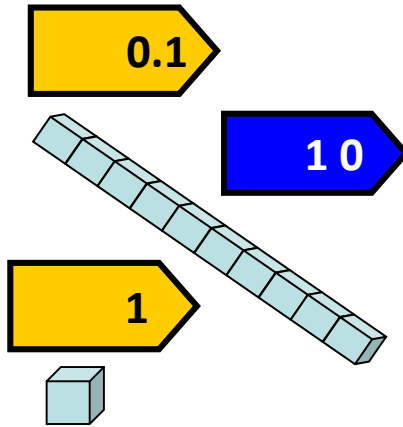
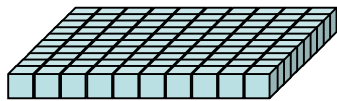
bead string



count stick

place value apparatus

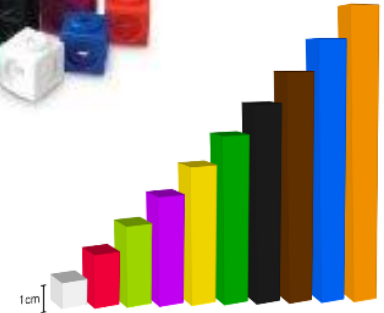
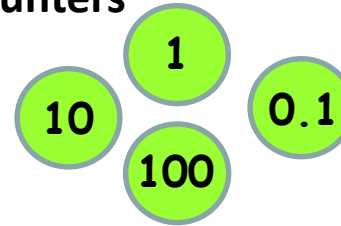
Hundreds	Tens	Units/Ones
 100s	 10s	 1s



Multilink

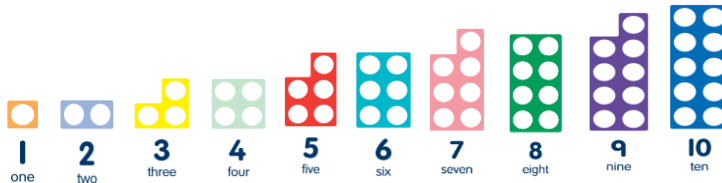


place value counters

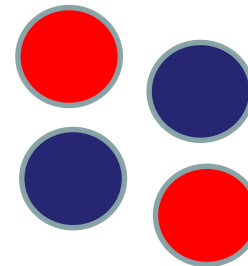
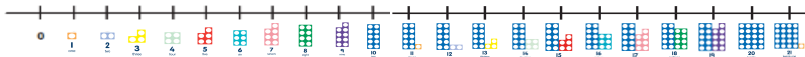


Cuisenaire

Numicon



number line



double sided counters

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

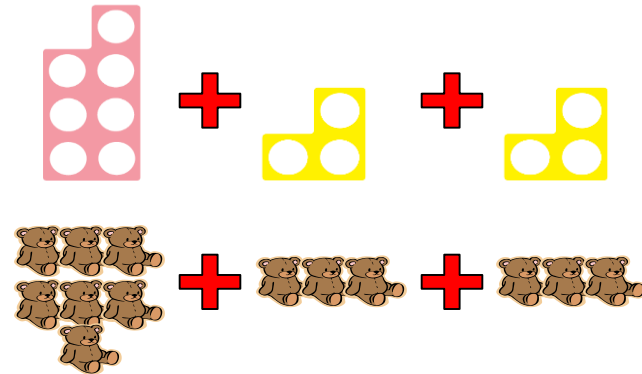
number grids
100 and 200

Structures of Addition (Haylock and Cockburn 2008)

Children should experience problems with all the different addition structures in a range of practical and relevant contexts e.g. money and measurement

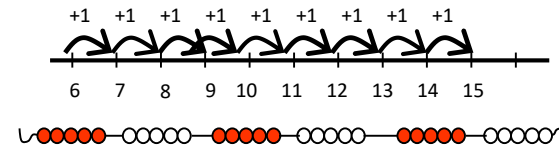
Aggregation

*Union of two sets
How many/much altogether?
The total*



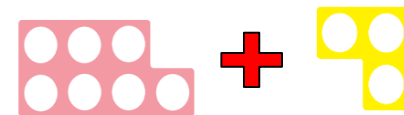
Augmentation

*Start at and count on
Increase by
Go up by*

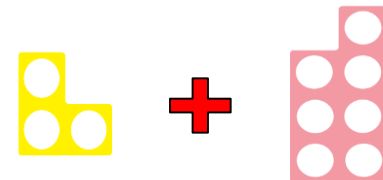


Commutative law

*Understand addition can be done in any order
Start with bigger number when counting on
(Explain to children that subtraction does not have this property)*



is the same as/equal to (=)



Addition

Pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly.
Addition and subtraction should be taught together.

End of Year Expectations

Year 1

U + U
TU + U

Numbers up to 20

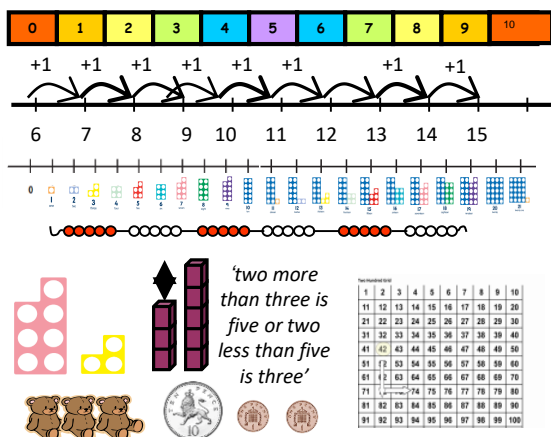
(including adding zero)

Children must experience combining two, **and then more than two**, groups of objects using counting on and the language of addition e.g. add, plus

Children must experience increasing numbers e.g. what is two more than seven?

Compare quantities to say how many less and/or how many more

Possible Concrete and Visual Representations



Use practical resources such as bears, counters, cubes and number lines/hundred grids and progress to a resource such as Numicon to encourage counting in groups rather than ones

Teacher Modelling/Children's Recording

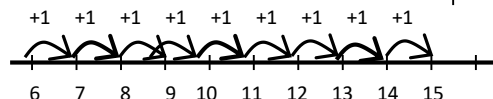
If using Numicon, children could use printed Numicon icons and stick these in - progressing to recording number sentences alongside

$$1 + 2 = 3$$

Children may record pictorially progressing to recording number sentences alongside

Example

6 and 9
6 + 9



Fluency

Count forwards, to and across 100, beginning with 0 or 1 or from any given number

Switch count between tens and ones e.g. 10, 20, 30, 31, 32, 33 ...

Represent and use number bonds up to 20 (establish addition and subtraction as related operations)

Find one more than a number

Find ten more than a number

Count in multiples of 2s, 5s and 10s starting on multiples to highlight pattern recognition

Vocabulary: count on, add, and, plus, more, sum, total, altogether,

Year 2

Children should be able to partition numbers in different ways e.g. as 2+2+2+1 or 5+3 or 23 as 20+3 or 10+13

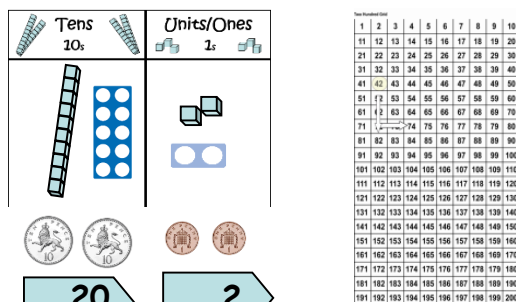
TU + U
TU + tens
TU + TU
U + U + U

Children should use concrete objects, pictorial representations and add numbers in different contexts e.g. money, measures

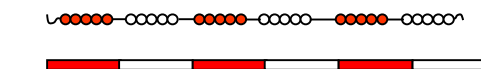
ENSURE CHILDREN HAVE THE OPPORTUNITY TO ADD **MORE** THAN TWO NUMBERS

Children should understand the language of sum

Ensure children understand that addition is commutative (can be done in any order)



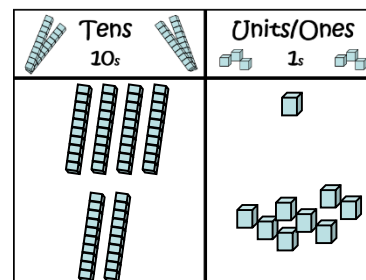
Numbered and partially numbered number lines



Use Numicon, number grids, place value apparatus/Dienes, place value grids, place value cards, Encourage children to partition numbers rather than counting in ones.

Children apply, develop and secure their understanding of place value

Use jottings and record number sentences



40

1

+ 20

+ 8

= 60

= 9

41

+

28

60 + 9 = 69

Show increasing fluency in deriving pairs of numbers up to 10 and then up to 20

Use knowledge to derive and use number facts up to 100

Add numbers mentally including TU + U, TU + tens, TU + TU, U + U + U

Vocabulary: count on, add, and, addition, plus, more, sum, total, altogether,

End of Year Expectations

Year 3

Add numbers with up to three-digits

(leading to formal written column method)

HTU + multiples of 10
HTU + multiples of 100

ENSURE CHILDREN HAVE THE OPPORTUNITY TO ADD **MORE THAN TWO NUMBERS** WITH DIFFERING NUMBERS OF DIGITS

Children should partition numbers, up to 1000, in different ways

e.g. $100 + 40 + 6$ or $100 + 30 + 16$

Solve problems in different contexts including missing number problems

Year 4

Add numbers with up to four-digits (formal written column method) including numbers with up to two decimal places in the context of money

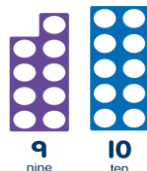
ENSURE CHILDREN HAVE THE OPPORTUNITY TO ADD **MORE THAN TWO NUMBERS** INCLUDING DECIMALS, WITH DIFFERING NUMBERS OF DIGITS

Solve two-step problems in different contexts including missing number problems

Possible Concrete and Visual Representations

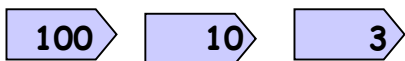
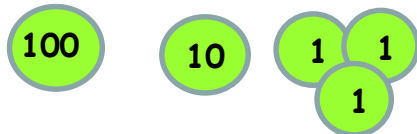
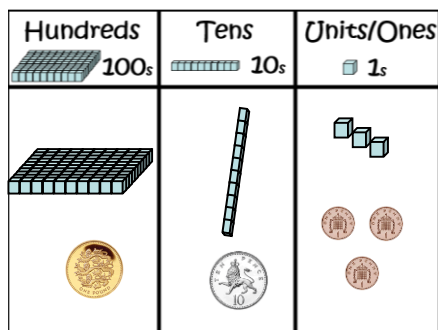
Two Hundred Grid

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
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51	52	53	54	55	56	57	58	59	60
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71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190
191	192	193	194	195	196	197	198	199	200

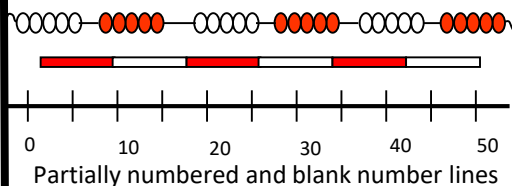
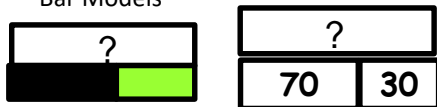


Place value chart

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



Bar Models



Teacher Modelling/Children's Recording

Children apply, develop and secure their understanding of place value and begin to record in columns

Manipulatives SHOULD be used alongside algorithms

Column addition (no exchanging) with up to three-digits

$$40 + 1$$

$$20 + 8$$

$$60 + 9 = 69$$

Expanded recording
without exchange

$$\begin{array}{r} 100 + 40 + 1 \\ 100 + 20 + 8 \\ 200 + 60 + 9 = 269 \end{array}$$

Expanded recording

$$40 + 3$$

$$20 + 8$$

$$\begin{array}{r} 60 + 11 \\ 10 + 1 = 71 \end{array}$$

Expanded recording
with exchange

$$\begin{array}{r} \text{HTU} \\ 141 \\ + 128 \\ \hline 269 \end{array}$$

Compact (column) recording

$$\begin{array}{r} 700 + 80 + 9 \\ 600 + 40 + 2 \\ 1300 + 120 + 11 = 1431 \end{array}$$

Column addition (with exchanging)

$$\begin{array}{r} \text{HTU} \\ 789 \\ + 642 \\ \hline 1431 \\ 11 \end{array}$$

Compact (column) recording

Add decimals in the
context of money

$$\begin{array}{r} \pounds 7.89 \\ + \pounds 6.42 \\ \hline \pounds 14.31 \\ 11 \end{array}$$

Fluency

Count in ones, tens and hundreds maintaining fluency through varied and frequent practice

Count from 0 in multiples of 4, 8, 50 and 100

Find 10 or 100 more than a number

Mentally add HTU + ones, HTU + tens, HTU + hundreds

Perform mental calculations with two-digit numbers, the answer could exceed 100

Vocabulary: count on, add, and, addition, plus, more, sum, total, altogether,

Count in 6s, 7s, 9s, 25s and 100s

Find 1000 more than a number

Perform mental calculations with increasingly large numbers to aid fluency

Vocabulary: count on, add, and, addition, plus, more, sum, total, altogether, increase

Addition

Pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly.
Addition and subtraction should be taught together.

End of Year Expectations

Year 5

Add numbers with more than four-digits and decimals up to three places
(formal written column method)

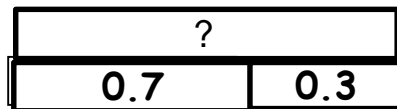
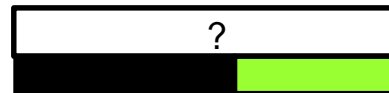
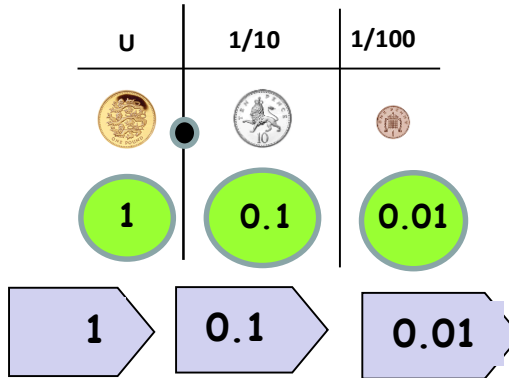
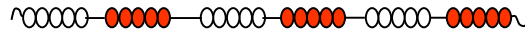
ENSURE CHILDREN HAVE THE OPPORTUNITY TO ADD **MORE THAN TWO NUMBERS** INCLUDING DECIMALS, WITH DIFFERING NUMBERS OF DIGITS

Solve addition (and subtraction) multi-step problems selecting and justifying methods

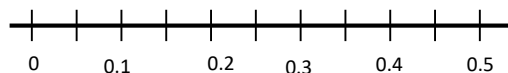
Practise mental calculations with increasingly large numbers

Possible Concrete and Visual Representations

0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	2	3	4	5	6	7	8	9



Bar Models



Partially numbered and blank number lines



Teacher Modelling/Children's Recording

Manipulatives could be used alongside algorithms

$$\begin{array}{r} 2141 \\ + 1128 \\ \hline 3269 \end{array}$$

Column addition (no exchanging)

$$\begin{array}{r} 21.41 \\ + 1.12 \\ \hline 0.35 \\ \hline 32.88 \end{array}$$

$$\begin{array}{r} 5189 \\ + 3128 \\ \hline 8317 \\ 11 \end{array}$$

Column addition (with exchanging)

$$\begin{array}{r} 51.89 \\ + 3.128 \\ \hline 55.018 \\ 11 \end{array}$$

Addition with decimals up to three decimal places including in different contexts e.g. money and measures

Fluency

Count forwards in powers of ten up to 100000

Count forwards in positive and negative whole numbers through zero

Practise mental calculations with increasingly large numbers

Practise fluency of written methods

Vocabulary: count on, add, and, addition, plus, more, sum, total, altogether, increase

Year 6

Add numbers with more than four-digits and decimals up to three places
(formal written column method)

ENSURE CHILDREN HAVE THE OPPORTUNITY TO ADD **MORE THAN TWO NUMBERS**, INCLUDING DECIMALS, WITH DIFFERING NUMBERS OF DIGITS

Solve more complex calculations mentally

Solve addition (and subtraction) multi-step problems in contexts, deciding which operations and methods to use and why

Count in tens and hundreds increasing fluency of order and place value

Perform increasingly complex mental calculations and those with increasingly large numbers to aid fluency

Vocabulary: count on, add, and, addition, plus, more, sum, total, altogether, increase

Structures of Subtraction (Haylock and Cockburn 2008)

Children should experience problems with all the different subtraction structures in a range of practical and relevant contexts e.g. money and measurement

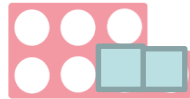
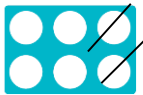
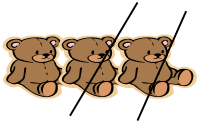
Partitioning

Take away

... how many left?

How many are not?

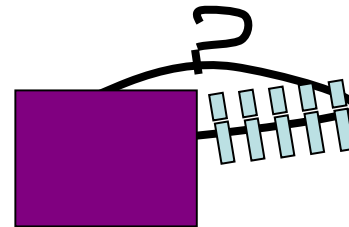
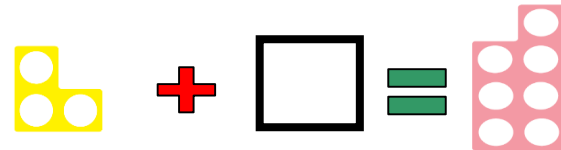
How many do not?



Inverse-of-addition

What must be added?

How many (much) more needed?



*There are ten pegs
on the hanger –
how many are covered?*

Comparison

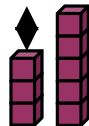
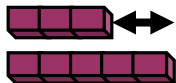
What is the difference?

How many more?

How many less (fewer)?

How much greater?

How much smaller?



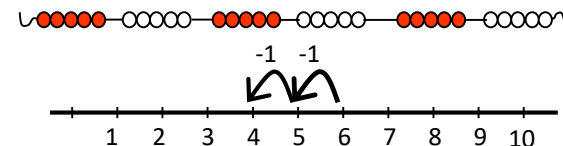
*'two more than three
is five or two less than
five is three'*

Reduction

Start at and reduce by

Count back by

Go down by



Subtraction

Pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly.

Addition and subtraction should be taught together.

End of Year Expectations

Year 1

U - U

TU - U

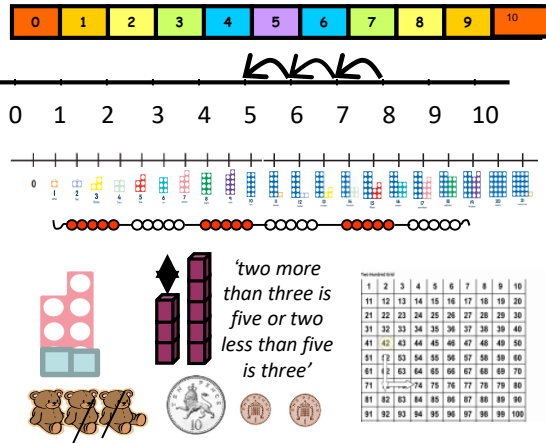
Numbers up to 20

(including subtracting zero)

Understand subtraction as taking away
What is ... less than ...?)

Compare quantities to say how many less and/or how many more

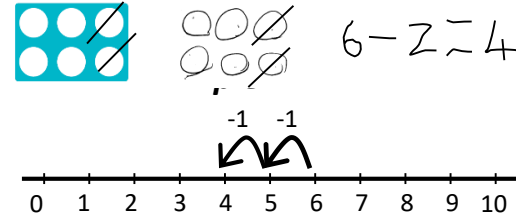
Possible Concrete and Visual Representations



Use practical resources such as bears, counters, cubes and number lines/hundred grids and progress to a resource such as Numicon to encourage counting back in groups rather than ones

Teacher Modelling/Children's Recording

Children may begin recording pictorially progressing to recording number sentences alongside



Children could use printed Numicon icons and stick these in, again progressing to recording number sentences alongside

Fluency

Count backwards (including crossing 100) any given number

Switch count between ones and tens e.g. 33, 32, 31, 30, 20, 10

Represent and use subtraction facts linked to number bonds up to 20 (establish addition and subtraction as related operations)

Find one less than a number

Find ten less than a number

Count back in multiples of 2s, 5s and 10s starting on multiples to highlight pattern

Vocabulary: leave, take away, fewer, subtract, minus, count back, difference between

Year 2

TU - U

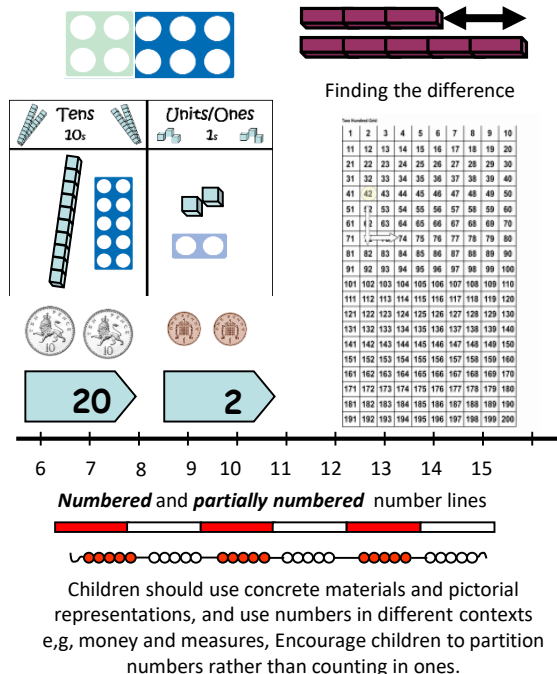
TU - tens

TU - TU

Understand subtraction as taking away and finding the difference

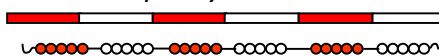
Ensure children understand that subtraction is not commutative (can not be done in any order)

Children should be able to partition numbers in different ways



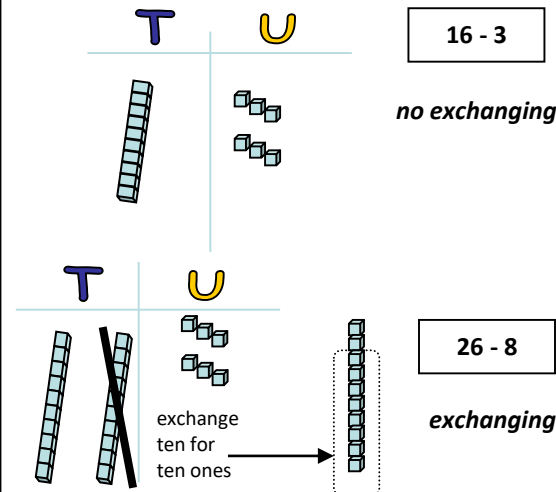
Finding the difference

Numbered and partially numbered number lines



Children should use concrete materials and pictorial representations, and use numbers in different contexts e.g, money and measures, Encourage children to partition numbers rather than counting in ones.

Children apply, develop and secure their understanding of place value and begin to record using jottings and number sentences



no exchanging

26 - 8

exchanging

Practise addition and subtraction facts to 20

Show increasing fluency in deriving subtraction facts for numbers up to 10 and then up to 20

Use known facts to 20 to derive new facts e.g. $3 + 7 = 30 + 70$

Use knowledge to derive and use subtraction number facts up to 100

Vocabulary: subtraction, leave, take away, fewer, subtract, minus, count back, difference between

Subtraction

Pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly.
Addition and subtraction should be taught together.

End of Year Expectations

Year 3

Subtract numbers with up to three-digits

(formal written column method)

HTU – U

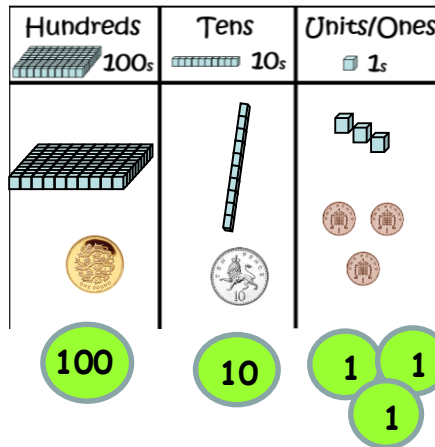
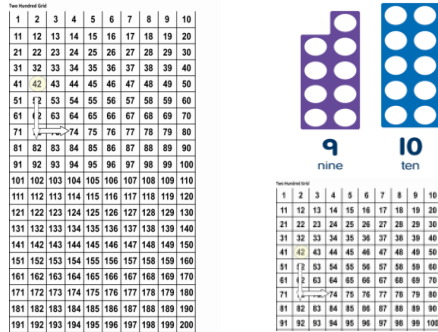
HTU – multiples of 10

HTU – multiples of 100

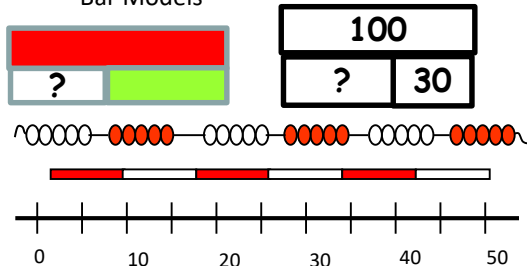
HTU – HTU

Children apply, develop and secure their understanding of place value and begin to record in columns

Possible Concrete and Visual Representations



Bar Models



Teacher Modelling/ Children's Recording

Children *SHOULD* use manipulatives alongside algorithms to transition between practical and abstract

no exchanging

$$68 - 23$$

$$\begin{array}{r} 60 \\ - 20 \\ \hline 40 \end{array}$$

$$40 \text{ and } 5 = 45$$

with exchanging

$$63 - 28$$

$$\begin{array}{r} 50 \quad 60 \quad 10 + 3 \\ - 20 \quad 8 \\ \hline 30 \end{array}$$

$$30 \text{ and } 5 = 35$$

Column subtraction (no exchanging)

$$148 - 121$$

$$\begin{array}{r} 100 \quad 40 \quad 8 \\ - 100 \quad 20 \quad 1 \\ \hline 20 \end{array}$$

$$20 \text{ and } 7 = 27$$

$$148$$

$$- 121$$

$$27$$

$$£1.48$$

$$- £1.21$$

$$£0.27$$

Column subtraction (with exchanging)

$$\begin{array}{r} 600 \quad 100 + 10 \\ - 700 \quad 20 \quad 10 + 3 \\ - 300 \quad 60 \quad 7 \\ \hline 300 \end{array}$$

$$300 \text{ and } 50 \text{ and } 6 = 356$$

$$723 - 367$$

$$\begin{array}{r} 6 \quad 11 \quad 1 \\ 7 \quad 2 \quad 3 \\ - 3 \quad 6 \quad 7 \\ \hline 3 \quad 5 \quad 6 \end{array}$$

$$\begin{array}{r} 6 \quad 11 \quad 1 \\ £7 \quad 2 \quad 3 \\ - £3 \quad 6 \quad 7 \\ \hline £3 \quad 5 \quad 6 \end{array}$$

Ensure children can solve calculations where zero is a place holder

Fluency

Count back in ones, tens and hundreds maintaining fluency through varied and frequent practice

Switch count between hundreds, tens and ones e.g 500, 400, 300, 290, 280, 270, 269, 268, 267

Mentally add HTU + ones, HTU + tens, HTU + hundreds

Perform mental calculations with two-digit numbers, the answer could exceed 100

Find ten and a hundred less than a number with up to three-digits

Vocabulary:

subtraction, leave, take away, fewer, subtract, minus, count back, difference between

Count back in 6, 7, 9, 25 and 1000

Count back through zero to include negative numbers

Find 1000 less than a number

Continue to practise mental calculations with increasingly large numbers to aid fluency

Vocabulary:

subtraction, leave, take away, fewer, subtract, minus, count back, difference between

Year 4

Subtract numbers with up to four-digits including up to two decimal places in the context of money

(formal written column method)

Understand subtraction as the inverse of addition

Solve two-step problems deciding upon the appropriate operations and methods and justifying choices made

Subtraction

Pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly.
Addition and subtraction should be taught together.

End of Year Expectations

Year 5

Subtract numbers with more than four-digits

Subtract numbers with up to three decimal places

Subtract larger numbers with more than four digits and those involving numbers up to three decimal places

(formal written column method)

ENSURE CHILDREN HAVE THE OPPORTUNITY TO ADD **MORE THAN TWO NUMBERS** INCLUDING DECIMALS, WITH DIFFERING NUMBERS OF DIGITS

Solve (addition) and subtraction multi-step problems selecting and justifying methods

Year 6

Subtract numbers with more than four-digits

Subtract numbers with up to three decimal places

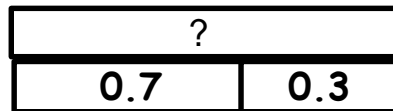
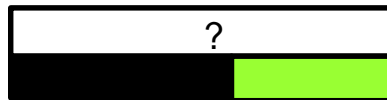
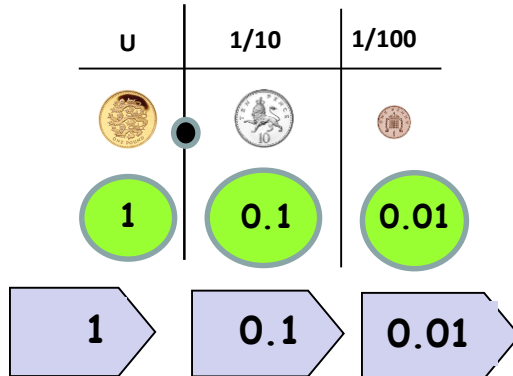
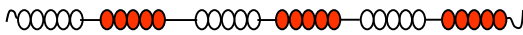
Subtract multi-digit numbers including numbers with up to three decimal places

(formal written column method)

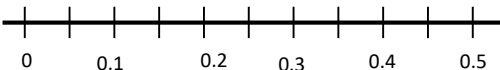
ENSURE CHILDREN HAVE THE OPPORTUNITY TO ADD **MORE THAN TWO NUMBERS** INCLUDING DECIMALS, WITH DIFFERING NUMBERS OF DIGITS

Solve (addition) and subtraction multi-step problems in contexts, deciding which operations and methods to use and why

0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	2	3	4	5	6	7	8	9



Bar Models



Teacher Modelling/ Children's Recording

Children might use manipulatives alongside algorithms

Column subtraction (no exchanging)

$$\begin{array}{r} 13548 \\ - 12128 \\ \hline 1420 \end{array}$$

Column subtraction (with exchanging)

$$\begin{array}{r} \overset{2}{1} \overset{13}{3} \overset{11}{4} \overset{1}{2} 3 \\ - 12678 \\ \hline 745 \end{array}$$

Ensure children can solve calculations where zero is a place holder

$$\begin{array}{r} 1.48 \\ - 1.21 \\ \hline 0.27 \end{array}$$

Column subtraction (no exchanging)

Column subtraction (with exchanging)

$$\begin{array}{r} \overset{6}{7} \overset{11}{.} \overset{1}{2} 3 \\ - 3.67 \\ \hline 3.56 \end{array}$$

Subtraction with decimals up to three decimal places including in different contexts e.g. money and measures

Fluency

Count backwards in powers of ten up to one million

Count backwards in positive and negative whole numbers through zero

Practise mental calculations with increasingly large numbers

Vocabulary:
subtraction, leave, take away, fewer, subtract, minus, count back, difference between

Undertake mental calculations with increasingly large numbers and more complex calculations

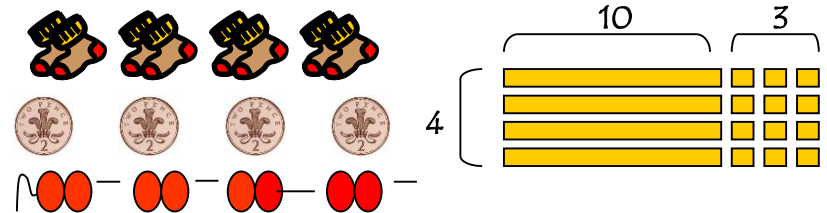
Vocabulary:
subtraction, leave, take away, fewer, subtract, minus, count back, difference between

Structures of Multiplication (Haylock and Cockburn 2008)

Children should experience problems with all the different multiplication structures in a range of practical and relevant contexts e.g. money and measurement

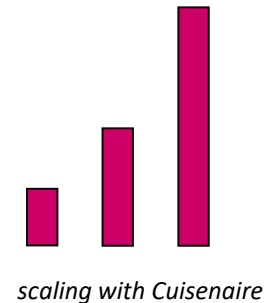
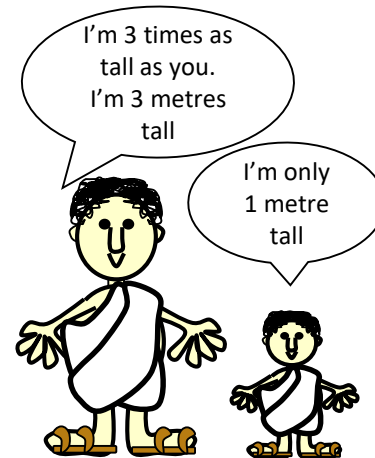
Repeated addition

*So many lots (sets) of so many
How many (how much) altogether
Per, each*



Scaling

*Scaling, scale factor
Doubling, trebling
So many times bigger than (longer than,
heavier than, and so on)
So many times as much as (or as many as)*



Commutative law

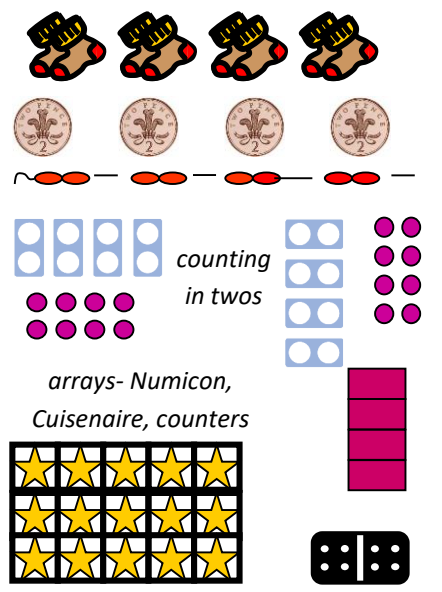
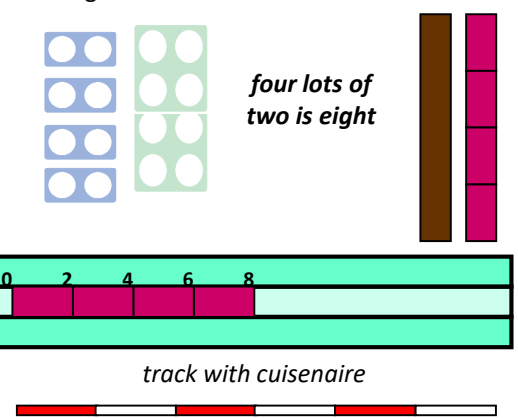
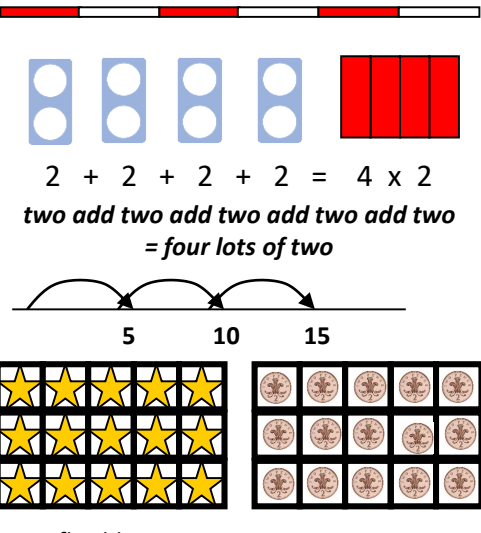
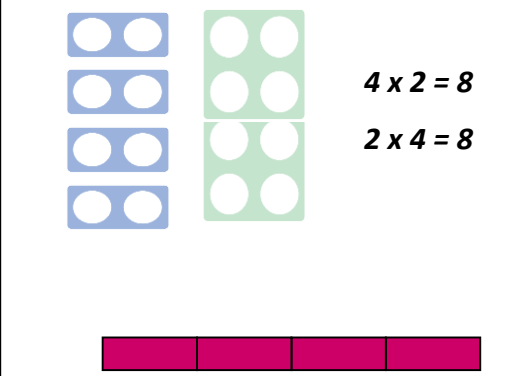
*Scaling, scale factor
Doubling, trebling
So many times bigger than (longer than,
heavier than, and so on)
So many times as much as (or as many as)*

a x b and b x a are equal

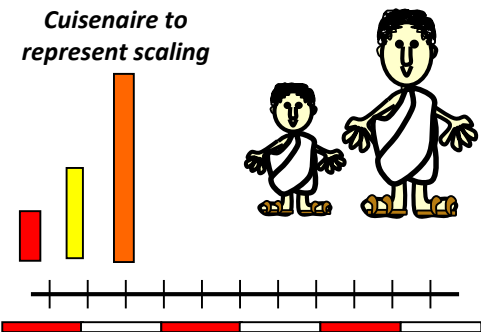
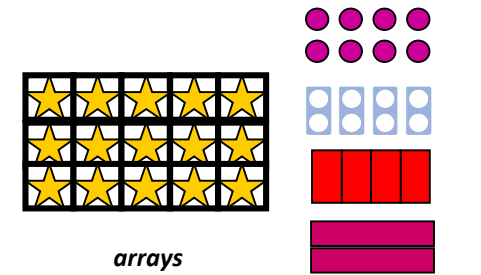
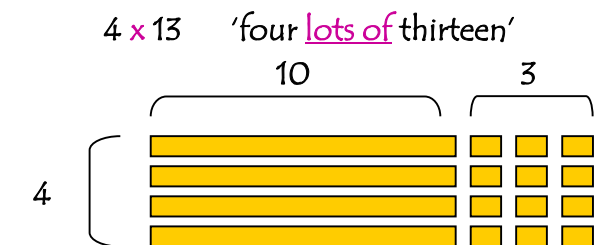

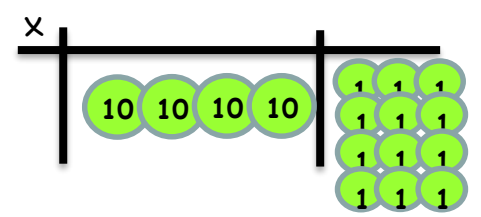
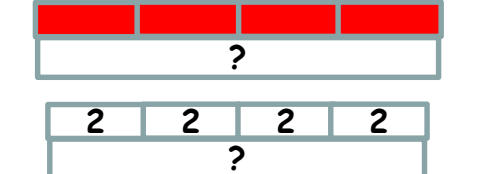


4 x 2 is the same as/equal to 2 x 4

Multiplication – refer to structures of multiplication

End of Year Expectations	Possible concrete and visual representation	Children's Recording	Fluency
<div data-bbox="19 199 193 271" data-label="Section-Header"> Year 1 </div> <div data-bbox="251 228 328 257" data-label="Text"> <p>U x U</p> </div> <p>Numbers up to 20</p> <p>Solve single step practical problems involving multiplication</p> <p>Use concrete objects, pictorial representations</p> <p>Make connections between arrays, number patterns and counting in twos, fives and tens</p> <p>Double numbers and quantities</p>	 <p>counting in twos</p> <p>arrays- Numicon, Cuisenaire, counters</p> <p>flexible array</p>	<p>Practical only e.g. link to small world</p> <p>Using concrete objects, pictorial representations and arrays with the support of an adult – take photographs/draw pictures – if using Numicon small icons could be stuck in</p>  <p>four lots of two is eight</p> <p>track with cuisenaire</p>	<p>Count in twos, fives and tens from different multiples</p> <p>e.g. 6, 8, 10, 12 etc</p> <p>Emphasise number patterns</p> <p>Double number and quantities</p> <div data-bbox="1593 606 1893 785" data-label="Text"> <p>Vocabulary: lots of, multiplied, double, groups of, array, multiply, times, multiplication</p> </div>
<div data-bbox="19 828 193 899" data-label="Section-Header"> Year 2 </div> <div data-bbox="270 842 347 871" data-label="Text"> <p>TU x U</p> </div> <p>Understand multiplication as repeated addition</p> <p>Calculate mathematical statements for multiplication within the tables and write them using symbols</p> <p>Understand and solve problems involving arrays</p> <p>Ensure children understand that multiplication is commutative (can be done in any order)</p> <p>Understand that multiplication and division are inverse operations</p>	 <p>two add two add two add two add two = four lots of two</p> <p>flexible array</p>	<p>Record practical work as number sentences</p>  <p>$4 \times 2 = 8$</p> <p>$2 \times 4 = 8$</p>	<p>Count in twos, threes, fives from zero and tens from any number</p> <p>e.g. 6, 8, 10, 12 etc</p> <p>Emphasise number patterns</p> <p>Introduction to multiplication tables. Practise to become fluent in multiplication facts for 2, 5 and 10</p> <p>Solve multiplication problems mentally</p> <div data-bbox="1593 1263 1893 1406" data-label="Text"> <p>Vocabulary: lots of, multiplied, double, groups of, array, multiply, times, multiplication</p> </div>

Multiplication – multiplication and division should be taught together– refer to structures of multiplication

End of Year Expectations	Possible concrete and visual representation	Teacher Modelling/Children’s Recording	Fluency								
<div>Year 3</div> <div>TU x U</div> <div>Develop reliable written methods</div> <div>Understand and solve scaling problems</div> <div>Solve problems involving multiplication including correspondence</div>	<div>Cuisenaire to represent scaling</div> <div></div> <div></div> <div>arrays</div>	<div>Children <u>must</u> use manipulatives alongside algorithms</div> <div>4×13 ‘four <i>lots of</i> thirteen’</div> <div></div> <div>Expanded methods – grid and area</div> <div><table><tr><td>x</td><td>10</td><td>3</td></tr><tr><td>4</td><td>40</td><td>12</td></tr></table></div> <div>$40 + 12 = 52$</div>	x	10	3	4	40	12	<div>Count from 0 in multiples of 4, 8, 50 and 100</div> <div>Use multiples of 2, 3, 4, 5, 8, 10, 50 and 100</div> <div>Practise mental recall of multiplication tables – 3, 4 and 8x times tables</div> <div>Connect the 2, 4 and 8 times tables using doubling</div> <div>Develop efficient mental methods using commutativity and multiplication facts to derive related facts e.g. $4 \times 4 \times 12 = 12 \times 4 \times 5 = 12 \times 20$</div> <div>Vocabulary: lots of, multiplied, double, groups of, array, multiply, times, multiplication, product</div>		
x	10	3									
4	40	12									
<div>Year 4</div> <div>TU x U</div> <div>HTU x U</div> <div>Multiplying three numbers</div> <div>Solve two-step problems</div> <div>Multiplying by 0 and by 1</div> <div>Develop fluency in short multiplication using formal written layout</div> <div>Solve problems involving multiplication including using the distributive law, integer scaling problems and harder correspondence problems</div>	<div></div> <div></div> <div></div> <div>bar models</div>	<div>Progressing to developing fluency in short multiplication</div> <div><table><tr><td>1 3</td><td>1 3 3</td></tr><tr><td>x 4</td><td>x 4</td></tr><tr><td>5 2</td><td>5 3 2</td></tr><tr><td>1</td><td>1 1</td></tr></table></div> <div>Start with digits that are below five so children can practise method without encountering difficulty with multiplication tables</div>	1 3	1 3 3	x 4	x 4	5 2	5 3 2	1	1 1	<div>Count in multiples of 6, 7, 9, 25 and 1000</div> <div>Recall and use multiplication facts up to 12 x 12 with increasing fluency</div> <div>Derive multiplication facts with up to three-digits</div> <div>Recognise and use factor pairs and commutativity in mental calculations</div> <div>Use the distributive law</div> <div>Combine knowledge of number facts and rules of arithmetic to solve mental and written calculations e.g. $2 \times 6 \times 5 = 10 \times 6$</div> <div>Vocabulary: lots of, multiplied, double, groups of, array, multiply, times, multiplication, product</div>
1 3	1 3 3										
x 4	x 4										
5 2	5 3 2										
1	1 1										

Multiplication - multiplication and division should be taught together– refer to structures of multiplication

End of Year Expectations

Year 5

Th HT U x U
HT U x TU
Th HT U x TU

Multiply decimals with up to three decimal places

Identify multiples and factors including finding all factor pairs of a number, and common factors of two numbers

Solve problems involving all four operations where larger numbers are used by decomposing them into their factors

Multiply whole numbers and those involving decimals by 10, 100 & 1000

Understand and use multiplication and division as inverses including in problems involving missing numbers and balancing equations

Solve problems involving multiplication and division including scaling by simple fractions

Know and use the vocabulary of prime numbers, prime factors and composite (non-prime)

Recognise and square and cube numbers and associated notation

Year 6

Multiply numbers up to 4-digit x TU

Multiply numbers with up to two decimal places x whole number

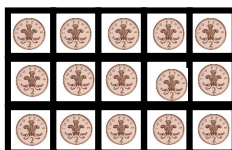
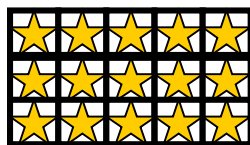
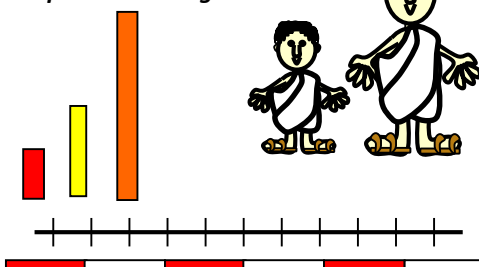
Multiply multi-digit numbers up to four-digits by a two-digit whole number

Multiply single –digit numbers with up to two-decimal places by whole numbers

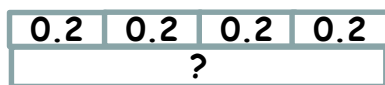
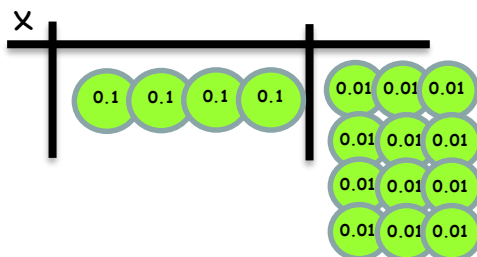
Solve problems involving all four operations

Possible concrete and visual representation

Cuisenaire to represent scaling



arrays



bar models

Teacher Modelling/Children's Recording

Children might use manipulatives alongside algorithms

Short multiplication

$$\begin{array}{r} 1324 \\ \times 6 \\ \hline 7944 \\ 112 \end{array}$$

Short multiplication

$$\begin{array}{r} 3.24 \\ \times 6 \\ \hline 19.44 \\ 12 \end{array}$$

Long multiplication

$$\begin{array}{r} 1324 \\ \times 26 \\ \hline 7944 \\ 26480 \\ \hline 34424 \\ 111 \end{array}$$

Long multiplication

$$\begin{array}{r} 3.24 \\ \times 26 \\ \hline 19.44 \\ 64.80 \\ \hline 84.24 \\ 11 \end{array}$$

Fluency

Count forwards in steps of powers of 10 from any given number up to 1 000 000

Practise and extend use of formal written method of short multiplication

Apply all multiplication tables frequently. Commit them to memory and use them confidently to make larger calculations

Multiply numbers mentally drawing upon known facts

Vocabulary: lots of, multiplied, double, groups of, array, multiply, times, multiplication, product

Undertake mental calculations with increasingly large numbers

Continue to use all multiplication tables to calculate mathematical statements in order to maintain fluency

Vocabulary: lots of, multiplied, product, double, groups of, array, multiply, times, multiplication

Structures for Division (Haylock and Cockburn 2008)

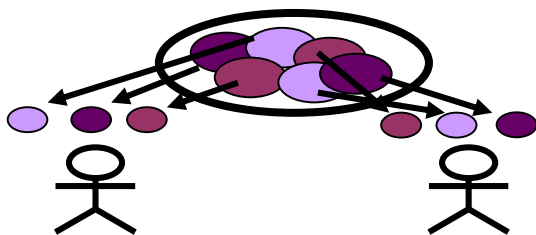
Children should experience problems with the different division structures in a range of practical and relevant contexts e.g. money and measurement

Equal-sharing

*Sharing equally between
How many (much) each?*

6 shared equally by **2**

$$6 \div 2$$

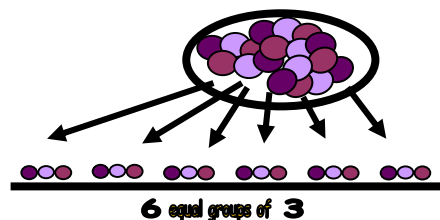


Inverse of multiplication (Grouping)

*So many lots (sets/groups) of so many
Share equally in to groups of ...*

$$18 \div 3$$

18 divided into
equal groups of **3s**



Divide twelve into equal
groups of four



= 3

Make 12

Overlay
groups of
four

Ratio structure






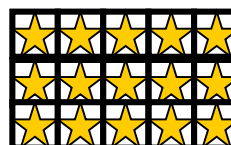
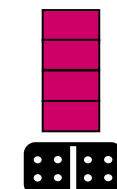

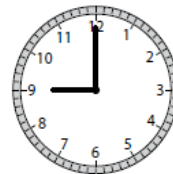


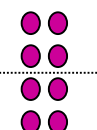


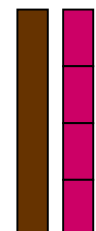
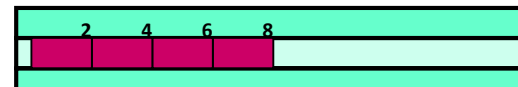

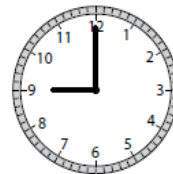


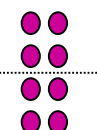

comparison

*inverse of scaling structure of multiplication
scale factor (decrease)*

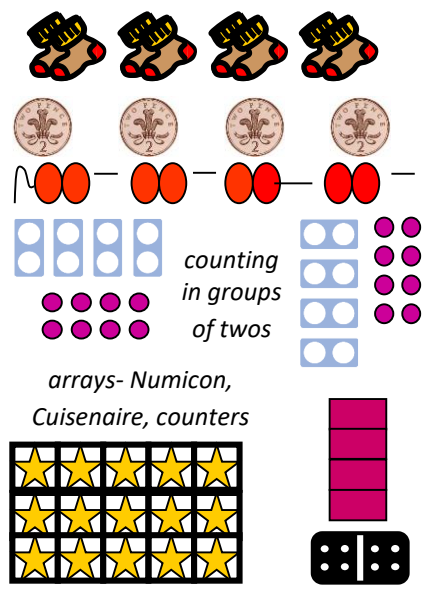
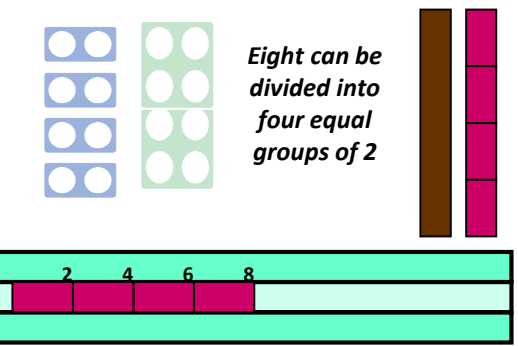
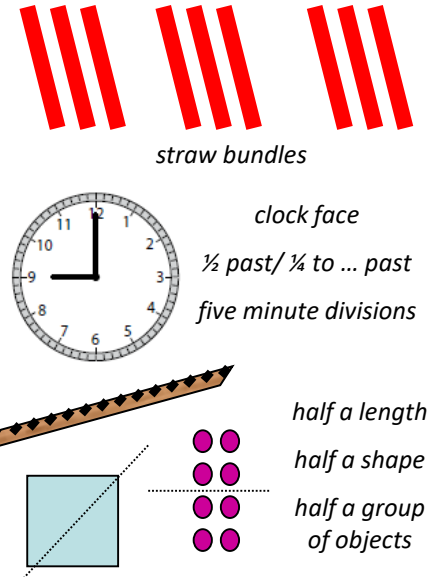

*Barney earns three times more than Fred. If
Barney earns £900 how much does Fred earn?*

*Jo's journey to school is three times as
long as Ella's. If Jo walks to school in
30 minutes how long does it take Ella?*

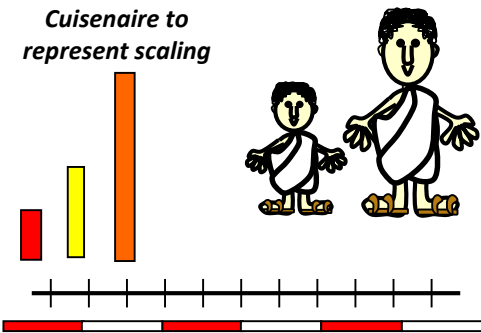

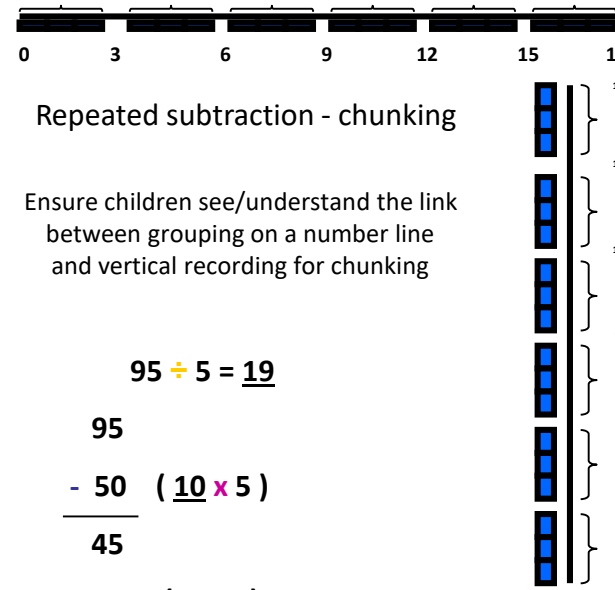

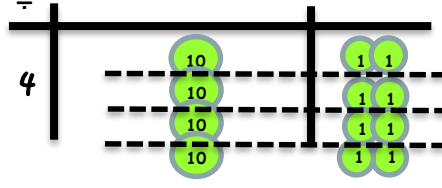
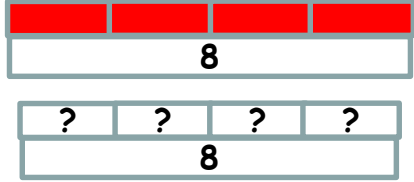
Division

End of Year Expectations	Possible concrete and visual representation	Teacher Modelling/Children's Recording	Fluency
Year 1 $U \div U$ Solve single step practical problems involving division Use concrete objects, pictorial representations Understand division as grouping and sharing Use the language of 'sharing equally between' Find halves and then quarters	    counting in groups of twos  arrays- Numicon, Cuisenaire, counters   <i>flexible array</i>  <i>straw bundles</i>  <i>clock face</i> <i>½ past/ ¼ to ... past</i> <i>five minute divisions</i>  <i>half a length</i>  <i>half a shape</i>  <i>half a group of objects</i>	Practical only e.g. link to small world Using concrete objects, pictorial representations and arrays with the support of an adult – take photographs/draw pictures – if using Numicon small icons could be stuck in   Eight can be divided into four equal groups of 2   <i>track with cuisenaire</i>	Count in twos, fives and tens from different multiples e.g. 6, 8, 10, 12 etc Emphasise patterns Double numbers and quantities Find simple fractions of objects, numbers and quantities
Year 2 $U \div U$ Solve single step practical problems involving division Use concrete objects, pictorial representations Understand division as grouping Find halves and then quarters Work with a range of materials and contexts in which multiplication and division relate to grouping and sharing discrete quantities e.g. marbles, sweets, cherries and continuous quantities e.g. cakes, pizzas, chocolate bars and relate to fractions and measures	 <i>straw bundles</i>  <i>clock face</i> <i>½ past/ ¼ to ... past</i> <i>five minute divisions</i>  <i>half a length</i>  <i>half a shape</i>  <i>half a group of objects</i>	Record as number sentences using \div and = $8 \div 4$ Eight divided into four equal groups = two in each group  $8 \div 4 = 2$	Count back in twos, threes, fives from zero and tens from any number e.g. 12, 10, 8, 6 etc Emphasise patterns Connect ten times table to place value and five times table to divisions on a clock face Introduction to multiplication tables. Practise to become fluent in division facts for 2, 5 and 10 Solve division problems involving grouping and sharing

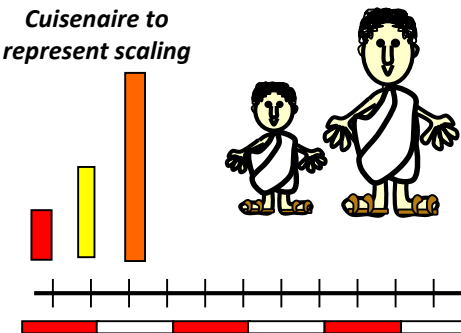
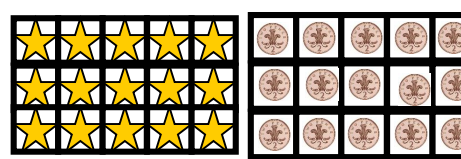

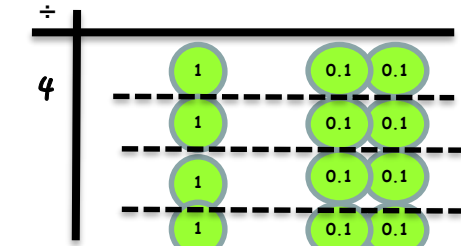
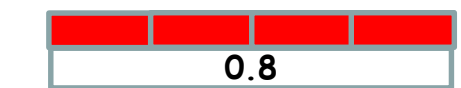
Division - multiplication and division should be taught together– refer to structures of division

End of Year Expectations	Possible concrete and visual representation	Children's Recording	Fluency
<div data-bbox="19 164 164 221" data-label="Section-Header"> <h2>Year 1</h2> </div> <div data-bbox="241 178 318 207" data-label="Equation-Block"> $U \div U$ </div> <p>Solve single step practical problems involving division</p> <p>Use concrete objects, pictorial representations</p> <p>Understand division as grouping and sharing</p> <p>Use the language of 'sharing equally between'</p> <p>Find halves and then quarters</p>	 <p>counting in groups of twos</p> <p>arrays- Numicon, Cuisenaire, counters</p> <p>flexible array</p>	<p>Practical only e.g. link to small world</p> <p>Using concrete objects, pictorial representations and arrays with the support of an adult – take photographs/draw pictures – if using Numicon small icons could be stuck in</p>  <p>Eight can be divided into four equal groups of 2</p> <p>track with cuisenaire</p>	<p>Count in twos, fives and tens from different multiples</p> <p>e.g. 6, 8, 10, 12 etc</p> <p>Emphasise patterns</p> <p>Find simple fractions of objects, numbers and quantities</p> <div data-bbox="1593 592 1893 778" data-label="Text"> <p>Vocabulary: equal groups of, divided by, lots of, divide, division, halve, half, share equally</p> </div>
<div data-bbox="19 852 164 909" data-label="Section-Header"> <h2>Year 2</h2> </div> <div data-bbox="260 866 338 895" data-label="Equation-Block"> $U \div U$ </div> <p>Solve single step practical problems involving division</p> <p>Use concrete objects, pictorial representations</p> <p>Understand division as grouping</p> <p>Find halves and then quarters</p> <p>Work with a range of materials and contexts in which multiplication and division relate to grouping and sharing discrete quantities e.g. marbles, sweets, cherries and continuous quantities e.g. cakes, pizzas, chocolate bars and relate to fractions and measures</p>	 <p>straw bundles</p> <p>clock face</p> <p>½ past/ ¼ to ... past</p> <p>five minute divisions</p> <p>half a length</p> <p>half a shape</p> <p>half a group of objects</p>	<p>Record as number sentences using \div and $=$</p> <p>$8 \div 4$</p> <p>Eight divided into four equal groups = two in each group</p>  $8 \div 4 = 2$	<p>Count back in twos, threes, fives from zero and tens from any number</p> <p>e.g. 12, 10, 8, 6 etc</p> <p>Emphasise patterns</p> <p>Connect ten times table to place value and five times table to divisions on a clock face</p> <p>Introduction to multiplication tables. Practise to become fluent in division facts for 2, 5 and 10</p> <p>Solve division problems involving grouping and sharing</p> <div data-bbox="1593 1278 1893 1406" data-label="Text"> <p>Vocabulary: equal groups of, divided by, lots of, divide, division, halve, half, share equally</p> </div>

Division - multiplication and division should be taught together– refer to structures of division

End of Year Expectations	Possible concrete and visual representation	Teacher Modelling/Children's Recording	Fluency
<div data-bbox="19 197 179 265"> Year 3 </div> <div data-bbox="216 215 282 237"> TU ÷ U </div> <p>Develop a reliable written method for division</p> <p>Solve problems involving missing numbers</p> <p>Solve problems including those that involve scaling</p> <p>Recognise, find and name $\frac{1}{2}$ and $\frac{1}{4}$ of an object, shape or quantity</p> <p>Understand the link between unit fractions and division</p> <p>Connect $1/10$ to division by 10</p> <p>Count in tenths</p>	<p><i>Cuisenaire to represent scaling</i></p>  <p><i>arrays</i></p> 	<p><i>Children should use manipulatives alongside algorithms</i></p>  <p>Repeated subtraction - chunking</p> <p>Ensure children see/understand the link between grouping on a number line and vertical recording for chunking</p> $95 \div 5 = \underline{19}$ $\begin{array}{r} 95 \\ - 50 \quad (\underline{10} \times 5) \\ \hline 45 \\ - 25 \quad (\underline{5} \times 5) \\ \hline 20 \\ - 20 \quad (\underline{4} \times 5) \\ \hline 0 \end{array}$ <div data-bbox="1255 885 1506 1120"> <p>Fact Box</p> <p>$2 \times 5 = 10$</p> <p>$5 \times 5 = 25$</p> <p>$10 \times 5 = 50$</p> </div>	<p>Recall and use related division facts for the 3, 4 and 8x tables (Continue to practise other tables)</p> <p>Write and calculate mathematical statements for division using what is known</p> <p>Use division facts to derive related division facts e.g. using $6 \div 3 = 2$ to work out $60 \div 3 = 20$</p> <div data-bbox="1613 549 1903 664"> <p>Vocabulary: equal groups of, divided by, lots of, divide, divisible by, factor, division, halve, half, share equally</p> </div>
<div data-bbox="19 768 179 836"> Year 4 </div> <div data-bbox="216 772 301 815"> TU ÷ U HTU ÷ U </div> <p>Become fluent in the formal written method of short division with exact answers when dividing by a one-digit number</p> <p>Divide one- or two-digit numbers by 10 or 100, identifying value of digits as tenths or hundredths</p> <p>Solve two-step problems in different contexts, choosing the appropriate operation, working with increasingly harder numbers including correspondence questions e.g. three cakes shared equally between 10 children</p>	<p>$48 \div 4$</p>  <p>$4 \overline{) 48}$</p>  <p>$4 \overline{) 48}$</p>  <p><i>bar models</i></p>	<p>Progressing to short division- no remainders</p> $560 \div 4$ $\begin{array}{r} \div 140 \\ 4 \overline{) 560} \\ \underline{4} \\ 16 \\ \underline{12} \\ 40 \\ \underline{40} \\ 0 \end{array}$ <p>See Appendix 1 – teaching short division with manipulatives</p>	<p>Continue to practise recalling division facts for multiplication tables up to 12×12</p> <p>Practise mental methods and extend this to three-digit numbers for example $200 \times 3 = 600$ into $600 \div 3 = 200$</p> <p>Use place value, known and derived facts to divide mentally, including dividing by 1</p> <p>Recognise and use factor pairs and commutativity in mental calculations</p> <div data-bbox="1613 1256 1903 1392"> <p>Vocabulary: equal groups of, divided by, lots of, quotient, divide, divisible by, factor, division, halve, half, share equally</p> </div>

Division - multiplication and division should be taught together– refer to structures of division

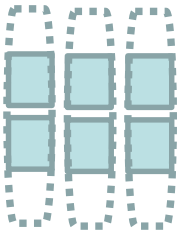
End of Year Expectations	Possible concrete and visual representation	Teacher Modelling/Children's Recording	Fluency
<div>Year 5</div> <p>Divide numbers with up to 4 digits by U</p> <p>Identify factors, including finding all factor pairs of a number, and common factors of two numbers</p> <p>Practise and extend the formal written method of short division: numbers up to four-digits by a one-digit number and interpret remainders appropriately for the context</p> <p>Interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding as appropriate for the context</p> <p>Use multiplication and division as inverses</p> <p>Solve problems involving division including scaling down by simple fractions and problems involving simple rates</p> <p>Divide whole numbers and those involving decimals by 10, 100 & 1000</p>	<p><i>Cuisenaire to represent scaling</i></p>  <p><i>arrays</i></p>  <p>$4.8 \div 4$</p>  <p>\div</p>   <p>bar models</p>	<p><i>Children might use manipulatives alongside algorithms</i></p> <div>Short division</div> <p>$564 \div 5$</p> <p>1 1 2 . 8</p> <p>5 $\overline{) 564.0}$</p> <p>remainder as a decimal</p> <p>See Appendix 1 – teaching short division with manipulatives</p> <div>Long division</div> <p>$560 \div 24$</p> <p>2 3 r 8</p> <p>24 $\overline{) 560}$</p> <p>- 48</p> <p>8 0</p> <p>- 72</p> <p>8</p> <p>remainder as a whole number</p> <p>$560 \div 24$</p> <p>2 3 8/24 (1/3)</p> <p>24 $\overline{) 560}$</p> <p>- 48</p> <p>8 0</p> <p>- 72</p> <p>8</p> <p>remainder as a fraction in its lowest form</p>	<p>Count backwards in steps of powers of 10 for any given number up to 1 000 000</p> <p>Count backwards with positive and negative whole numbers through zero</p> <p>Practise mental calculation with increasingly large numbers</p> <p>Apply all multiplication tables and related division facts frequently, commit them to memory and use them to confidently to make larger calculations</p> <div>Vocabulary: groups of, divided by, lots of, left over, quotient, divide, divisible by, factor, remainder, division, halve, half, share</div> <p>Practise division for larger numbers, using the formal written methods of short and long division</p> <p>Continue to use all multiplication tables and division facts to maintain fluency</p> <p>Perform mental calculations, including with mixed operations and larger numbers</p> <div>Vocabulary: groups of, divided by, lots of, left over, quotient, divide, divisible by, factor, remainder, division, halve, half, share</div>
<div>Year 6</div> <p>Divide numbers with up to 4 digits by TU</p> <p>Divide decimals up to two-decimal places by U or TU</p> <p>Divide numbers up to 4-digits by a 2-digit whole number using formal written methods of long division, interpret remainders as whole numbers, fractions or by rounding, as appropriate for the context</p> <p>Divide numbers with up to 2 decimal places by 1-digit and 2-digit whole numbers, initially in practical contexts involving money and measures</p> <p>Understand the relationship between unit fractions and division</p> <p>Recognise division calculations as the inverse of multiplication</p> <p>Solve problems involving division</p>			

Moving to written algorithms

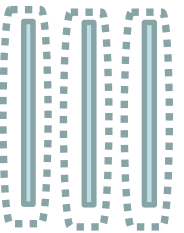
Short Division – no exchange

$$\begin{array}{r|l} \div & 212 \\ \hline 3 & 636 \end{array}$$

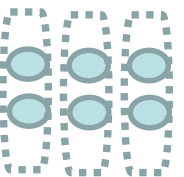
Divide 6 hundreds into 3 equal groups
How many in each group?



Divide 3 tens into 3 equal groups
How many in each group?



Divide 6 ones into 3 equal groups
How many in each group?

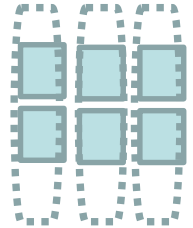


Moving to written algorithms

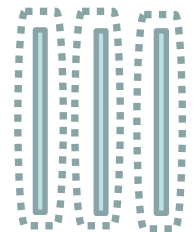
Short Division – with exchange

$$\begin{array}{r|l} \div & 215 \\ \hline 3 & 64^15 \end{array}$$

Divide 6 hundreds into 3 equal groups
How many in each group?



Divide 4 tens into 3 equal groups
How many in each group?
Exchange the remaining ten into ten ones



Divide 15 ones into 3 equal groups
How many in each group?

